ProTerrain: Probabilistic Physics-Informed Rough Terrain World Modeling

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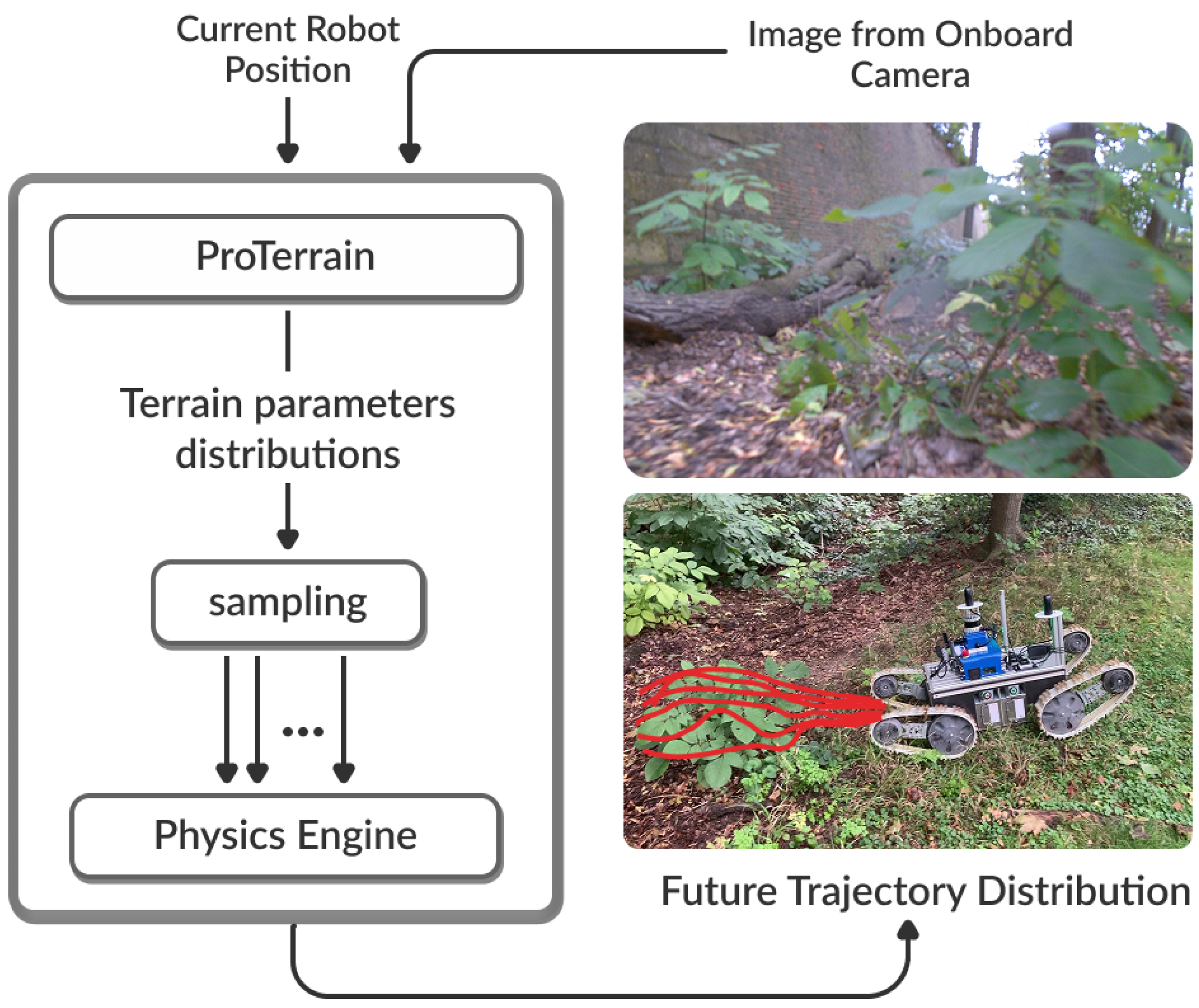
Motivation

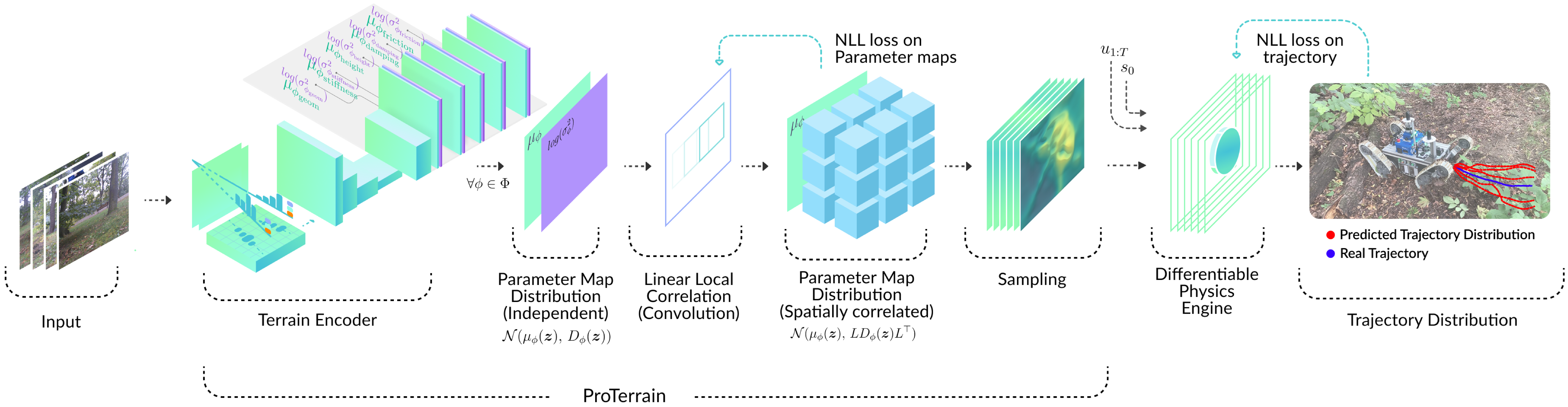
- Reliable planning calls for **physics-informed** models that explicitly models robot-terrain interaction.
- **Safe, risk-aware off-road autonomy** requires **uncertainty-aware** motion prediction and traversability assessment in **heterogeneous, ambiguous** terrain.
- Prior approaches assume either **deterministic** maps or **per-cell independent** uncertainty, neglecting **spatial correlation** in 3D terrain and producing unrealistic risk estimates.
- **Explicit high-dimensional covariance** over terrain is **computationally infeasible** at practical map resolutions.



Goals and Contributions

- Design an end-to-end **probabilistic terrain world model** with **spatially correlated aleatoric uncertainty** for robust off-road navigation.
- Ensure tractable, high-resolution learning using a scalable **structured covariance** based on convolutional operations.
- Achieve uncertainty-aware, **probabilistic trajectory forecasts** by propagating uncertainty through a **differentiable physics engine**.

Model Architecture



Problem Formulation

- World model: estimate spatially correlated terrain parameters ϕ (e.g., elevation, support, friction) as:

$$W_t = \{p(\phi|\mathrm{z})\}_{\phi \in \Phi}, \quad p(\phi|\mathrm{z}) = \mathcal{N}(\mu_\phi(\mathrm{z}), \Sigma_\phi(\mathrm{z}))$$

$$\mathcal{L}_{\text{NLL}} = \frac{1}{2}\left[\log\det\Sigma(z) + r^\top \Sigma(z)^{-1} r\right]$$

- Probabilistic trajectory prediction:

$$p\left(\tau | W_t, s_0, \mathrm{u}_{1:T}\right) = \mathcal{N}(\mu_\tau(\cdot), \Sigma_\tau(\cdot))$$

Structured Spatial Aleatoric Uncertainty

- Spatial uncertainty: **structured multivariate Gaussian** with covariance matrix:

$$\Sigma(\mathrm{z}) = LDL^\top, \; D = \operatorname{diag}(\sigma^2), \; L = \operatorname{Toeplitz}(g)$$

- Leads to efficient closed-form negative log-likelihood (NLL) loss:

$$\mathcal{L}_{\text{NLL}} = \frac{1}{2}\sum_{i=1}^{n}\left(b_i^2 + \log\sigma_i^2\right)$$

with $\quad \Sigma \mathrm{a} = \mathrm{r}, \; \mathrm{b} = D^{1/2}L^\top \mathrm{a}$

- All mat-vec operations are implicit via convolution:

$$L\,\mathrm{vec}(X) = \mathrm{vec}(g * X), \quad \mathrm{x} \mapsto LDL^\top \mathrm{x} \Leftrightarrow g * [\sigma^2 \odot (g^\top * X)]$$

Probabilistic World Model

- From onboard images, predict spatial maps of terrain parameters (e.g., elevation, friction, stiffness, damping).
- Use our convolutional structured covariance to capture spatially correlated uncertainty.
- Produces a full probabilistic terrain world model for robust, uncertainty-aware planning.

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Probabilistic Trajectory Forecast

- **Simulate robot dynamics** as:

$$\dot{\mathrm{x}} = \mathrm{v}, \quad \dot{\mathrm{v}} = \frac{1}{M}\sum_i \mathrm{f}_i, \quad \dot{R} = [\omega]R, \quad \dot{\omega} = J^{-1}\sum_{i=1}^{N}(\mathrm{p}_i \times \mathrm{f}_i)$$

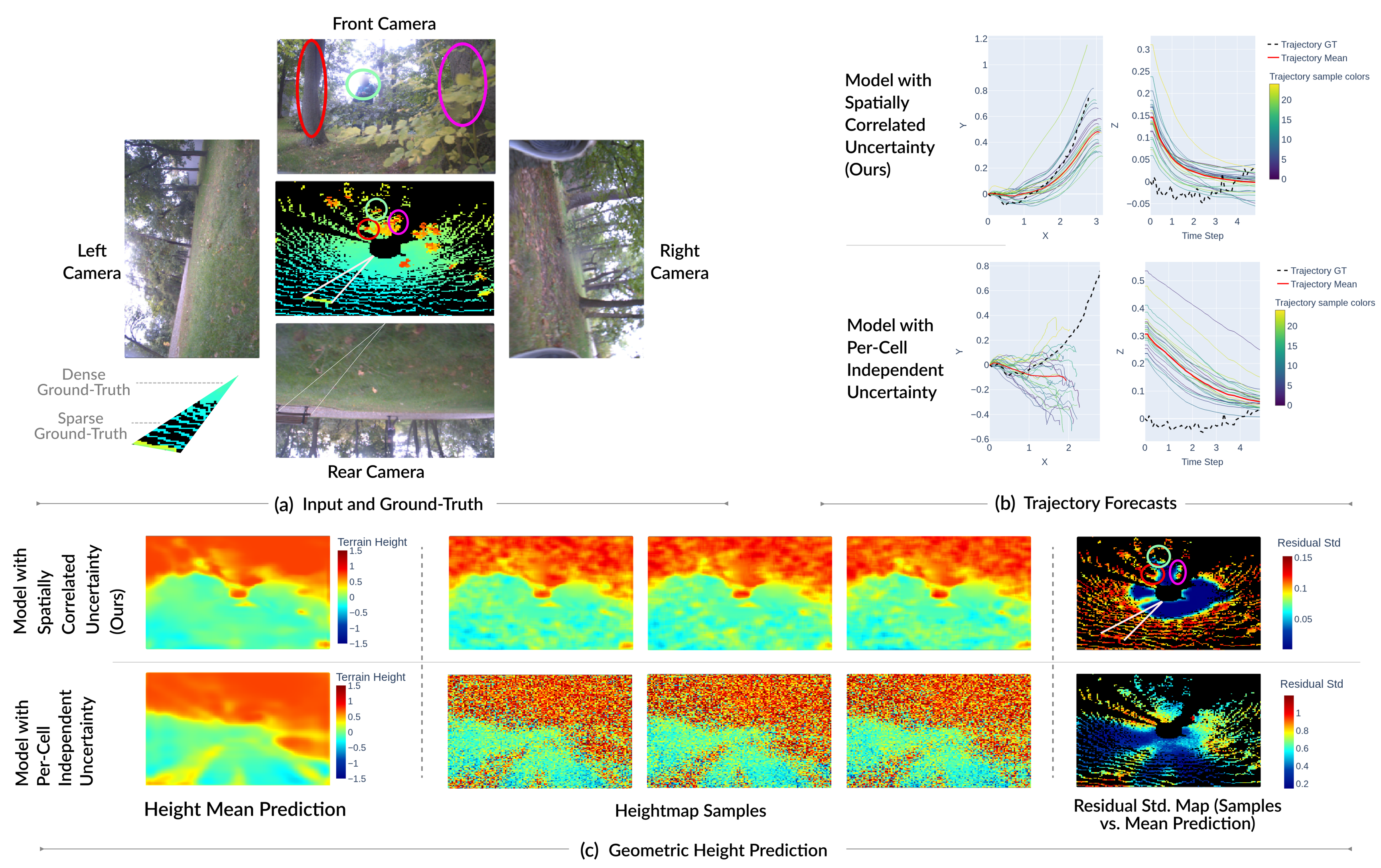
- **Sample terrain parameters** from the probabilistic world model:

$$\mathrm{vec}(\phi^{(m)}) = \mathrm{vec}(\mu_\phi) + L\sqrt{D_\phi}\,\epsilon^{(m)}, \quad \epsilon^{(m)} \sim \mathcal{N}(0,1)$$

- **Forecast robot trajectories** by simulating motion for each terrain sample via differentiable physics — yielding a probabilistic trajectory distribution.

Results

- Improved trajectory accuracy and uncertainty calibration.
- Uncertainty maps highlight ambiguous/occluded areas.
- Realistic, diverse map and trajectory samples closely match ground truth.



Disclaimer: This work has been submitted to ICRA 2026 and is under review.

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